Year 12 Mathematics Specialist 2018

Test Number 3: Vectors

Resource Rich

Name: _____ **Teacher: DDA**

Marks: **45**

Time Allowed: **45 minutes**

Instructions: You are permitted 1 A4 page of notes and your calculator. Show your working where appropriate remembering you must show working for questions worth more than 2 marks.

Question 1 **[1, 2, 3 = 6 marks]**

If $\boldsymbol{a} = < -2,3,1 >$ and $\boldsymbol{b} = < 3,1,-5 >$ find:

a) $-\boldsymbol{a} - 5\boldsymbol{b}$

b) The size of the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

c) The acute angle between $\boldsymbol{a}$ and the $x - y$ plane.

Question 2 **[2 mark]**

Find the vector equation of the line perpendicular to the plane $2x+3y-z=5$ and that contains the point $P(1,-2,0)$.

Question 3 **[2 marks]**

Find the vector equation of a plane that contains the line $\mathbf{r}(t)=\begin{pmatrix}1\\2\\-1\end{pmatrix}+t\begin{pmatrix}3\\0\\2\end{pmatrix}$ and the point $P(-1,2,-4)$.

Question 4 **[1,1,3,2 = 7 marks]**

Two parallel planes have the following equations

Plane Π: $\boldsymbol{r}.\begin{pmatrix}2\\-3\\6\end{pmatrix} = 14$ Plane Ω: $\boldsymbol{r}.\begin{pmatrix}2\\-3\\6\end{pmatrix} = 42.$.

a) Point A with position vector $4\boldsymbol{i} + 2\boldsymbol{j} + c\boldsymbol{k}$ lies on the plane Π. Find the value of c.

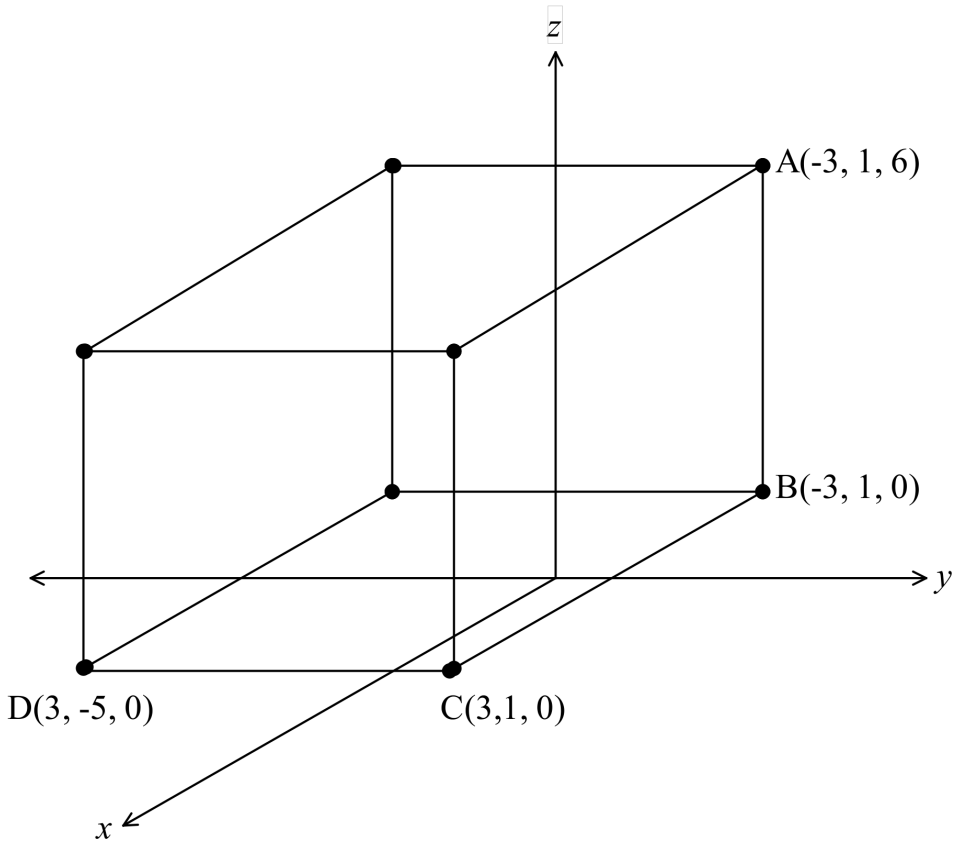
b) Determine the equation of the line L that passes through A and is perpendicular to plane Π.

c) Determine the position vector of B, the point of intersection of line L with plane Ω.

d) Determine the exact distance between the planes Π and Ω.

Question 4 **[3 marks]**

Find the equation of a sphere that fits exactly inside the cube on the diagram below.



Question 6 **[1,1, 3 = 5 marks]**

A little boy, holding a sandwich in his hand at (0, 0, 0.5), is running along the street such that the position vector of the sandwich is $\boldsymbol{r}(t) = \begin{pmatrix} 0 \\ 0 \\ 0.5 \end{pmatrix} + t \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$ where t is measured in seconds from $t = 0$.

A kookaburra at $(-5.5, -1.5, 4.5)$ eyed off the sandwich for one second then swooped down with a velocity of $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ to pinch the sandwich.

(a) Show that the position vector of the kookaburra from $t = 1$ is $\boldsymbol{r}_k(t) = \begin{pmatrix} -7.5 \\ -2.5 \\ 5.5 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(b) How fast did the kookaburra fly? Distances are measured in metres.

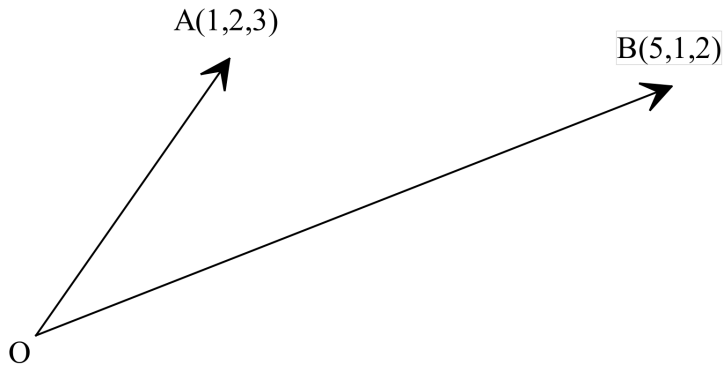
(c) How many seconds does the kookaburra take to steal the sandwich (not including the second when the bird is eyeing off the sandwich).

Question 7 **[3, 2 = 5 marks]**

(a) OABC is a parallelogram with OA parallel to CB. Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$.

Prove that the area of the parallelogram OABC is $|\boldsymbol{a} \times \boldsymbol{c}|$.

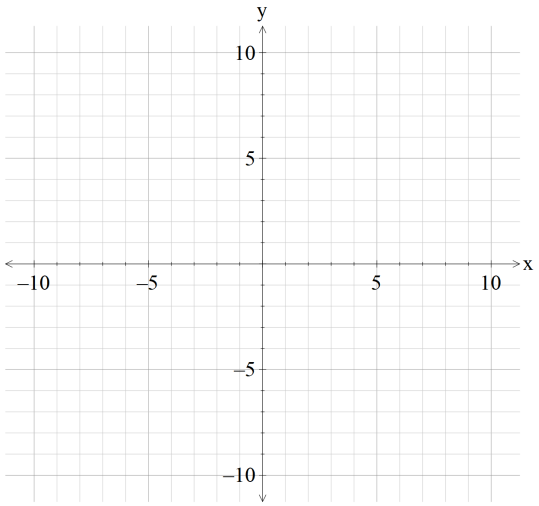
(b) Hence, show the use of vectors methods to determine the area of the triangle AOB in the diagram below.



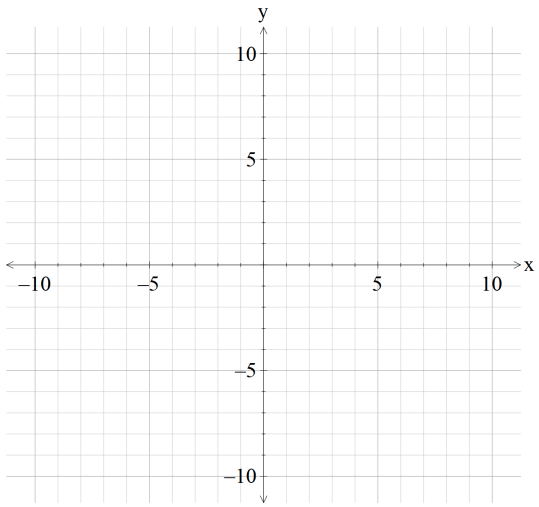
Question 8 **[2, 2, 3 = 7 marks]**

Find the Cartesian equation of the path traced by the point P with position vector $\boldsymbol{r}(t)$, where t represents time. Sketch the path, indicating starting position and the direction of motion.

a) $\boldsymbol{r}(t) = \begin{pmatrix} t \\ 4 \end{pmatrix}$



b) $\boldsymbol{r}(t) = \begin{pmatrix} 7\sin t + 1 \\ 7\cos t \end{pmatrix}$



c) Show algebraically how the vector equation in b) could be converted to the Cartesian equation.

Question 9 **[3, 3, 2 = 8 marks]**

A particle P is projected from the origin with a speed of 60 ms^{-1} at an angle of 30° to the horizon. Assume that the only force acting on P is the gravitational force, 9.8 ms^{-2}.

a) Find an expression for the position vector of P t seconds after projection.

b) Find the time taken for P to reach its maximum height and hence find the time of flight (the time the particle is in the air).

c) Find the horizontal displacement of P.